Exam 1. Math 4541

Friday, October 5, 2018 12:34 PM

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Last name:
First name:
BU or BG:

Problem 1 (5 points). For the differential equation, $x' = x^4 - x^2$, find all equilibrium solutions and determine whether they are sinks, sources, or neither. Also sketch the phase line.

Problem 2 (5 points). The family of differential equations $x' = x^3 - x + a$ depends on a parameter a. Sketch the corresponding bifurcation diagrams.

Problem 3 (5 points). Consider the differential equation $x' = f(t,x)$, where $f(t,x)$ is continuously differentiable in t and x. Suppose that $f(t+T,x) = f(t,x)$ for all t and x . Suppose there are constants p and q such that $p < q$ and $f(t,p) > 0$ and $f(t,q) < 0$ for all t. Prove that there is a periodic solution $x(t)$ for this equation with $p < x(0) < q$.

Problem 4 (5 points). Solve the initial value problem

$$x_1' = -x_1 + x_3 \qquad x_1(0) = 1$$
$$x_2' = -x_2 - x_3 \qquad x_2(0) = -1$$
$$x_3' = -x_3 \qquad x_3(0) = 2$$

Problem 5 (5 points). Solve the initial value problem

$$x_1' = x_1 + 2x_2 \qquad x_1(0) = 1$$
$$x_2' = -2x_1 + x_2 \qquad x_2(0) = 2$$